Orthogonalized Infinite Edge Element Method –Convergence Improvement by Orthogonalization of Hilbert Matrix in Infinite Edge Element Method–

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Abstract — This paper proposes an orthogonalization of Hilbert matrix in element matrices of the infinite edge elements. The Hilbert matrix results in extremely slow convergence in the ICCG method when we used higher order expansion. The orthogonalization improves the convergence drastically and it makes the infinite elements practical in the electromagnetic FEM analysis of the open boundary problems. A numerical example demonstrates the effectiveness of the proposed method.

I. INTRODUCTION

The electromagnetic phenomena intrinsically spread over the infinite space. Thus, the efficient handling of open boundary is one of the essential issues in the electromagnetic field computations [1-3]. In this paper, a new formulation of the infinite element combined with the usual FEM is proposed as a novel effective method for open boundary problems.

In previous researches, we investigated the infinite element formulation with the expansion function from the viewpoint of the multipole expansion to distant potentials [4][5]. This paper proposes a new method of the infinite element based on the sequence of orthogonal expansion functions, which results in the drastic reduction of CPU-time; an advantage of the proposed infinite edge element technique. With the new formulation, the orthogonalized infinite edge element method can become a powerful tool for magnetic field analysis. To demonstrate the effectiveness of the proposed method, a numerical example is presented.

II. FORMULATION OF THE ORTHOGONALIZED INFINITE EDGE ELEMENT METHOD

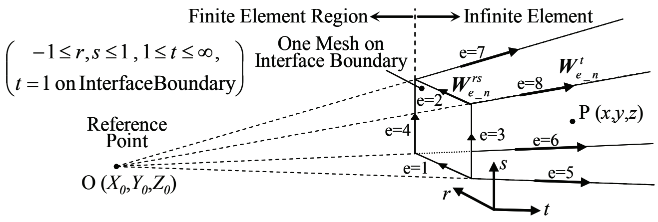

Fig. 1. Vector shape functions in the infinite edge element.

The infinite element in this paper possesses a shape formed by edges radiated into infinite space from the interface boundary. The diagrammatic illustration of the infinite edge element is shown in Fig. 1. The infinite edge element is projected out into infinite space from an arbitrary point O with the coordinates (X_0,Y_0,Z_0) as the reference point, and is composed of eight edges: four edges tangential to the interface boundary and four edges radiating out into infinite space. The four edges at infinity can be eliminated by the Dirichlet boundary condition where the magnetic vector potential $\boldsymbol{A}$=0.

The unknown variables $(A^{rs}_{e\_n}, A^{t}_{e\_n})$ are superposed on each edges of the infinite element, where the subscript e represents the edge number, and n is the expansion order.

The local coordinate system of the infinite edge element is defined with the rst coordinate axes as shown in Fig. 1. The local coordinate values of the r and s axes take the values between -1 and 1. By contrast, the t-axis is defined in the radial direction from the reference point, with the value of t =1 on the interface boundary, and taking the range of 1 to ∞. For example, the x-coordinate of an arbitrary point P inside the infinite edge element can be given by the following equation

$$x = X_0 + \sum_i \omega_i(r,s)\,t(x_i - X_0) \quad (1)$$

Here, ω_i represents a 2-D scalar shape function,

$$\omega_i(r,s) = \frac{1}{4}(1 + r_i r)(1 + s_i s) \quad (2)$$

related to each nodal point on the rs-axes of the interface boundary. The scalar field inside the infinite edge element can be interpolated with the scalar shape function,

$$N_i(r,s) = \omega_i(r,s)\varphi_n(t) \quad (n = 1,2,\cdots,N) \quad (3)$$

where $\varphi_n(t)$ is the scalar expansion function.

The vector shape functions $(\boldsymbol{W}^{rs}_{e\_n}, \boldsymbol{W}^{t}_{e\_n})$ of the infinite edge element can be represented based on the gradient expression of the scalar shape function in (3). Using the vector shape functions of each edge, the magnetic vector potential in the infinite edge element can be derived:

$$\begin{aligned}\boldsymbol{A} &= \sum_{n=1}^{N}\left(\sum_{e=1}^{4}\boldsymbol{W}^{rs}_{e\_n}A^{rs}_{e\_n} + \sum_{e=5}^{8}\boldsymbol{W}^{t}_{e\_n}A^{t}_{e\_n}\right) \\ &= \sum_{n=1}^{N}\left[\sum_{e=1}^{4}\varphi_n(t)\{f_e(r,s)\nabla r + g_e(r,s)\nabla s\}A^{rs}_{e\_n} - \sum_{e=5}^{8}\frac{d\varphi_n(t)}{dt}h_e(r,s)\nabla t A^{t}_{e\_n}\right]\end{aligned} \quad (4)$$

where f_e and g_e are the sum of (2) related to the two nodal points, i.e, both ends of the investigated edge on the interface boundary, and h_e is (2) related to the nodal point at the origin of the edge radiating out into infinite space.

In [4][5], we adopted the following expansion function,

$$\varphi_n(t) = \frac{1}{t^n} \tag{5}$$

from the viewpoint of the multipole expansion to distant potentials. The conventional formulation based on (5), however, generates the local matrices with the same form as a part of the Hilbert matrix. This results in the poorly-converged characteristic of the ICCG method, especially in the case of high expansion orders [5].

To overcome the above difficulty, in this paper, we newly derive the sequence of orthogonal expansion functions $\varphi_{\perp n}(t)$ as shown in the following equation,

$$\int_1^{\infty} \frac{\varphi_{\perp n}(t)\varphi_{\perp m}(t)}{t^2} dt = 0 \quad (n \neq m) \tag{6}$$

and propose the novel formulation of infinite edge element based on $\varphi_{\perp n}(t)$. The details of the functions and the formulation, in which we carry out a further orthogonalization, will be reported in the full paper.

III. NUMERICAL EXAMPLE

Changing the series expansion order of the infinite edge element, we analyzed a single coil in a vacuum shown in Fig. 2. The interface boundary between the finite and infinite element regions is also shown in Fig. 2.

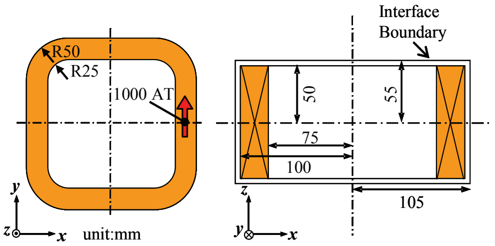

Fig. 2. Single coil model.

We analyzed the model using the infinite edge element formulated with the orthogonal expansion function $\varphi_{\perp n}(t)$ in (6) and compared the magnetic flux density B_z along the z-axis with theoretical values obtained from the Biot-Savart law [6]. The results are shown in Fig. 3. Fig. 3 indicates that the higher we set the series expansion order, we are able to obtain a sufficiently accurate result in spite of the interface boundary placed significantly close to the coil.

Fig. 4 shows the convergence characteristics of the ICCG method in the cases of the formulations with the expansion functions in (5) and (6). In both cases, the higher the series expansion order becomes, the number of required iterations is inclined to increase. However, we can confirm that the proposed formulation fairly improve the convergence characteristics in comparison with the conventional one. The comparison of the computational burden is also shown in Table I.

From Figs. 2-4 and Table I, it can be said that the proposed orthogonalized infinite edge element method enables us to drastically reduce the computational burden with keeping the analysis' accuracy. The full paper will report some numerical examples demonstrating the effectiveness of the proposed method.

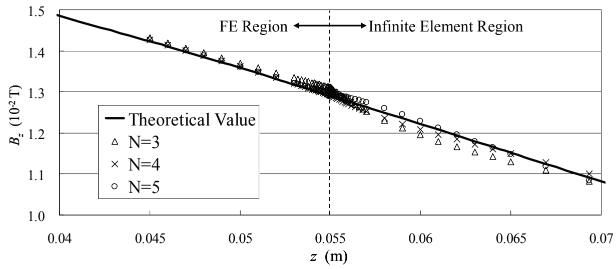

Fig. 3. Magnetic flux B_z around the interface boundary (Condition 1).

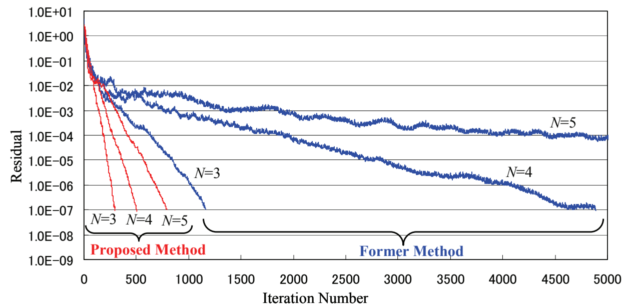

Fig. 4. Comparison of convergence characteristics

Table I Comparison of computational burden.

Analysis Method		Number of Elements	Iteration Number	CPU-Time (s)
Infinite Element Method Order N=5	Former	34,496	17,523	1,552.13 (9.55)
	Orthogonalized		794	71.89 (0.44)
Usual FEM $0 \le x,y,z \le 1000$	$\begin{pmatrix}\text{BC of} \\ Bn=0\end{pmatrix}$	531,831	203	162.46 (1.00)

IV. REFERENCES

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